Linear Collider Tests of Grand Unified Theories

James Wells
University of Michigan

January 2004

Discussion based on work done with Kazuhiro Tobe in preprint hep-ph/0312159.

Supersymmetric Unification

Simple idiomatic statement: "The gauge couplings unify in supersymmetry!"

A more complete statement: "The gauge couplings renormalize to within about 1/2 percent at a very high scale, which is what one expects would happen if the SM gauge groups result from a spontaneously broken $SU(5)$-like simple group."

There is no exact unification of the renormalized gauge couplings of the low-scale effective theory, nor should there be.

Supersymmetric grand unified theories have GUT-scale states that must be addressed if one wants to get down to brass tacks about unification.

SUSY and Grand Unification

Simplicity: SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is an equal-rank subgroup of the simpler $SU(5)$.

Particle content: All known particles fit very nicely within complete $SU(5)$ representations $\bar{\mathbf{5}}$ and $\mathbf{10}$.

Anomaly cancellation: Subtle cancellations of anomalies of the $U(1)_Y$ are trivially understood by a symmetry if the $U(1)_Y$ comes from spontaneous symmetry breaking of $SU(5)$ and all particles fit in complete representations of $SU(5)$.

Gauge coupling unification: Supersymmetric theories show precision gauge coupling to within expected "high-scale slop" (less than 1 percent).

Stability of Scales: Supersymmetry has a built-in symmetry mechanism to protect low-mass particles from having their masses raised to the GUT scale.

GUT-scale corrections in Minimal $SU(5)$

Nearby the GUT scale the MSSM couplings relate to the GUT scale couplings by

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_G^2(Q)} + \Delta_i^G(Q)$$

where,

$$\begin{aligned}
\Delta_1^G(Q) &= \frac{1}{8\pi^2}\left(-10\ln\frac{Q}{M_V} + \frac{2}{5}\ln\frac{Q}{M_{H_c}}\right) \\
\Delta_2^G(Q) &= \frac{1}{8\pi^2}\left(-6\ln\frac{Q}{M_V} + 2\ln\frac{Q}{M_\Sigma}\right) \\
\Delta_3^G(Q) &= \frac{1}{8\pi^2}\left(-4\ln\frac{Q}{M_V} + \ln\frac{Q}{M_{H_c}} + 3\ln\frac{Q}{M_\Sigma}\right).
\end{aligned}$$

Triplet Higgsino Mass Constraint

Define the scale Λ_U to be where $g_1(\Lambda_U) = g_2(\Lambda_U) = g_U$. This is the "GUT scale" and g_U is the "GUT gauge coupling". It is unambiguously determined from low-energy physics. One finds

$$1 \times 10^{16}\,\mathrm{GeV} \lesssim \Lambda_U \lesssim 2 \times 10^{16}\,\mathrm{GeV}$$

for superpartners at the TeV scale and below.

From previous equations we find

$$\frac{1}{g_U^2} - \frac{1}{g_3^2(\Lambda_U)} = \frac{3}{10\pi^2} \ln \frac{M_{H_c}}{\Lambda_U}.$$

It is well known that $g_3(\Lambda_U) < g_U$, albeit by less than 1%. (Forcing $g_3(\Lambda_U) = g_U$ and then running down to weak scale gives $\alpha_s(m_Z) \gtrsim 0.125$, which is too high.)

Thus, the LHS is negative. For the RHS to be negative and match, $m_{H_c} < \Lambda_U$. In other words, exact unification in minimal $SU(5)$ appears to require

$$m_{H_c} \lesssim 10^{16}\,\mathrm{GeV}$$

Minimal $SU(5)$ and the Higgsino triplet

The heavy Higgsino triplet accompanying the Higgs doublet in the 5 representation is a problem for proton decay.

From proton decay studies: $m_{H_c} > 10^{17}\,\mathrm{GeV}$ required.

From gauge coupling unification studies: $m_{H_c} \simeq \mathrm{few}\times 10^{15}\,\mathrm{GeV}$. (Getting the "slop" right.)

These observations lead to papers such as

"Not even decoupling can save minimal SUSY SU(5)"
Murayama and Pierce, Physical Review D, 2002

Enabling minimal $SU(5)$

There is a very natural path $SU(5)$ unification (JW & Tobe), which surfaces if we consider gravity corrections to the gauge kinetic function

$$\mathcal{L} = \int d^2\theta \left[\frac{1}{g^2} \mathcal{W}^a \mathcal{W}^a + \frac{\Sigma_{ab}}{M_{\rm Pl}} \mathcal{W}^a \mathcal{W}^b + \cdots \right]$$

where Σ is the 24-adjoint GUT-Higgs field and $\mathcal{W}^a$ contains the GUT-gauge field.

We expect

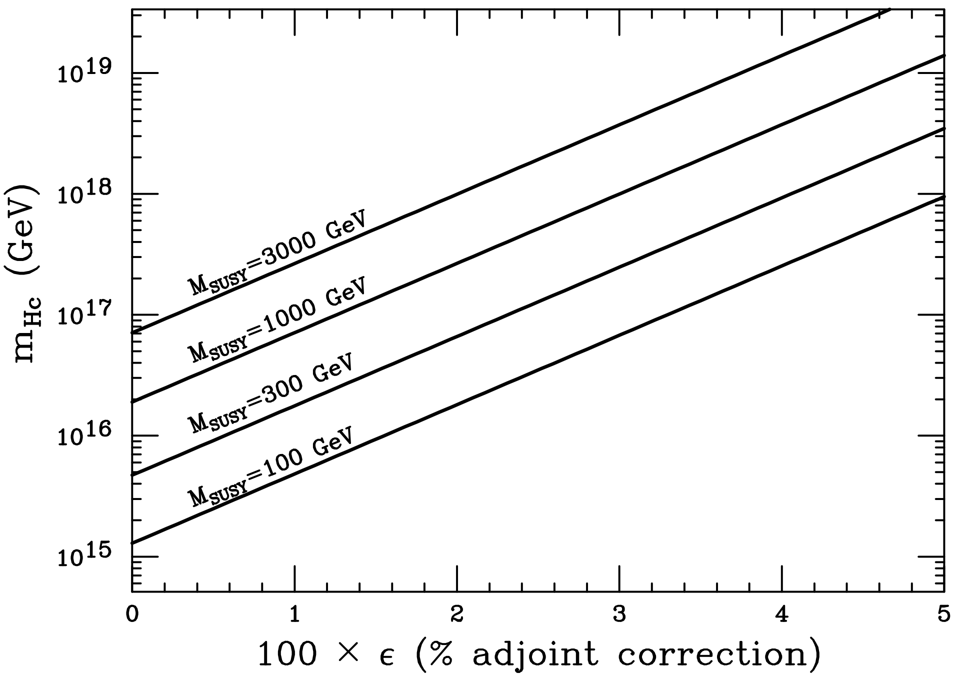
$$\frac{\langle \Sigma_{ab} \rangle}{M_{\rm Pl}} = c_i \epsilon \quad \text{where}$$

$$c_i = \{-2, -3, 1\} \;\; \text{and} \;\; \epsilon \sim \frac{M_{\rm GUT}}{M_{\rm Pl}} \sim \text{few}\,\%$$

Thus, gauge couplings get shifted very slightly by

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_G^2} + \Delta_i^{\rm loop}(Q) + c_i \epsilon$$

Adjoint-Gravity corrections in $SU(5)$ unification



Observable consequences: The low-scale gaugino masses ($\tilde{\gamma}$, $\tilde{Z}$, and $\tilde{W}^{\pm}$) are altered by "superspace vev":

$$\langle \hat{\Sigma}_{ab} \rangle = \langle \Sigma_{ab} \rangle + \theta^2 \langle F_{ab} \rangle$$

which leads to shifts of the gaugino mass

$$M_i' = M_i + \frac{\langle F_{ab} \rangle_i}{M_{\mathrm{Pl}}} = M_i(1 + \mathcal{O}(1)\, c_i \epsilon)$$

Gaugino Mass Corrections

The superpotential and soft lagrangian terms we assume are

$$W = \frac{1}{2} M_\Sigma \mathrm{Tr}\, \Sigma^2 + \frac{f}{3} \mathrm{Tr}\, \Sigma^3 + M_5 \mathbf{5}_H \overline{\mathbf{5}}_H + \lambda \overline{\mathbf{5}}_H \Sigma \mathbf{5}_H + \cdots$$

$$\begin{aligned} -\mathcal{L}_{\rm soft} \; = \; & \frac{1}{2} B_\Sigma M_\Sigma \mathrm{Tr}\, \Sigma^2 + \frac{f}{3} A_\Sigma \mathrm{Tr}\, \Sigma^3 + B_5 M_5 \mathbf{5}_H \overline{\mathbf{5}}_H \\ & + A_\lambda \lambda \overline{\mathbf{5}}_H \Sigma \mathbf{5}_H + h.c. + \cdots \end{aligned}$$

where upon minimizing the full potential we find

$$F_\Sigma \simeq v_\Sigma (A_\Sigma - B_\Sigma) = \frac{\epsilon M_{\rm Pl}}{8y} (A_\Sigma - B_\Sigma)$$

which generates a correction to gaugino masses via the operator

$$\int d^2\theta \, \frac{y \Sigma}{M_{\rm Pl}} \mathcal{W}\mathcal{W}$$

where the superfield vev of Σ is

$$\langle \hat{\Sigma} \rangle \simeq (v_\Sigma + F_\Sigma \theta^2) \, \mathrm{diag} \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, -1, -1 \right)$$

Gaugino Masses

After some computations one finds

$$
\begin{aligned}
M_1(\Lambda_U) &= g_U^2\overline{M} + g_U^2\left[\frac{1}{6}\epsilon(A_\Sigma - B_\Sigma) - \frac{1}{16\pi^2}\left(10g_U^2\overline{M} + 10\{A_\Sigma - B_\Sigma\} + \frac{2}{5}B_5\right)\right] \\
M_2(\Lambda_U) &= g_U^2\overline{M} + g_U^2\left[\frac{1}{2}\epsilon(A_\Sigma - B_\Sigma) - \frac{1}{16\pi^2}\left(6g_U^2\overline{M} + 6A_\Sigma - 4B_\Sigma\right)\right] \\
M_3(\Lambda_U) &= g_3^2(\Lambda_U)\overline{M} + g_U^2\left[-\frac{1}{3}\epsilon(A_\Sigma - B_\Sigma) - \frac{1}{16\pi^2}\left(4g_U^2\overline{M} + 4A_\Sigma - B_\Sigma + B_5\right)\right]
\end{aligned}
$$

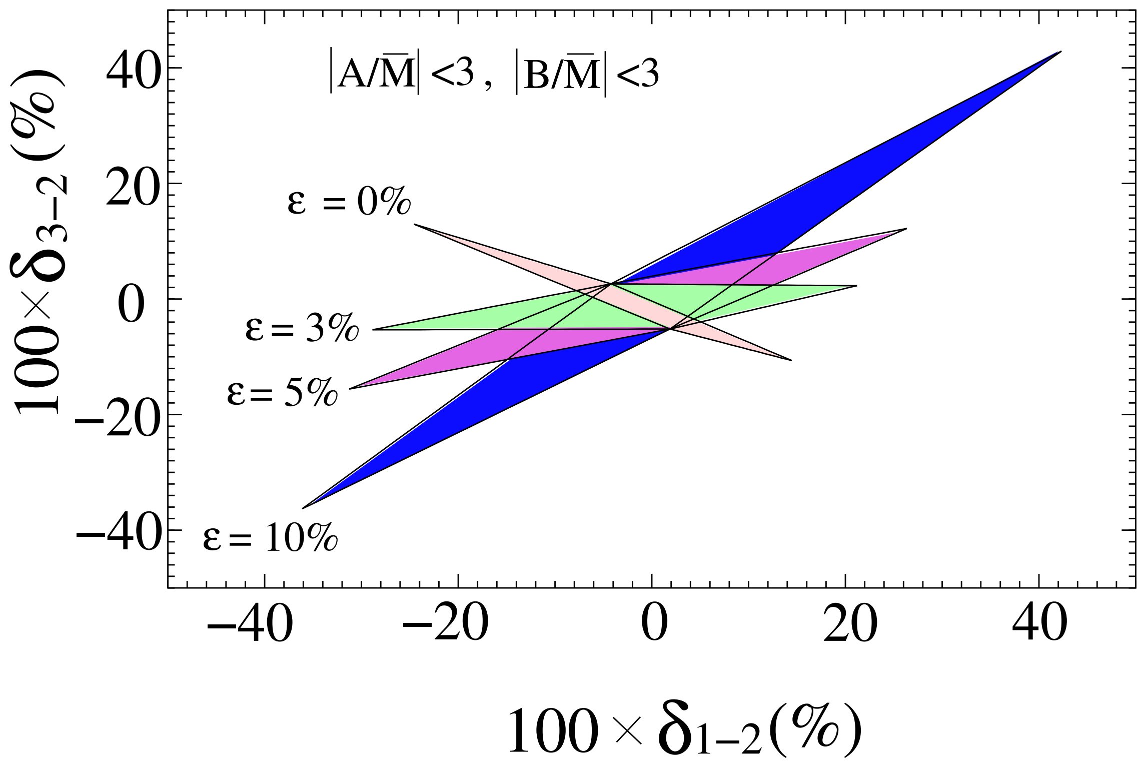
The overall scale of gaugino masses cannot be predicted. Focus on the ratio of gaugino masses

$$\delta_{1-2} = \frac{M_1(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)} \quad \text{and} \quad \delta_{3-2} = \frac{M_3(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)}.$$

One solution to the doublet-triplet splitting problem requires universal A terms and B terms. The ratios are then described by three dimensionless parameters

$$\epsilon, \quad A/\overline{M}, \quad B/\overline{M}.$$

ϵ needs to be a few percent from gauge coupling unification. We will choose $|A/\overline{M}|$ and $|B/\overline{M}|$ to range from -3 to $+3$.



- Large corrections to gaugino masses are possible from GUT-scale effects.
- Non-zero ϵ has a significant effect on the gaugino mass ratios.
- Further abstractions, such as assuming measured A terms at low scale are equal to universal A terms at GUT scale, etc., could allow determination of ϵ.
- *In any event, GUT-scale ideas would rise and fall from precise measurements of gaugino masses.*

Colliders and GUT gaugino masses

The combination of Linear Collider and LHC data could enable very precise determinations of the gaugino masses at the GUT scale.

For example, Blair, Porod, and Zerwas, hep-ph/0210058, demonstrate in two of their tables that $M_1(\Lambda_U)$ and $M_2(\Lambda_U)$ could be extrapolated to a fraction of a percent and $M_3(\Lambda_U)$ could be extrapolated to within a couple of percent.

Corrections to gaugino masses of more than a few percent would be measurable at the colliders.

Careful measurement of gaugino masses directly tests interesting ideas of grand unification.