Implications of Little Higgs Models

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With Spencer Chang, hep-ph/0311177, PLB(2004)

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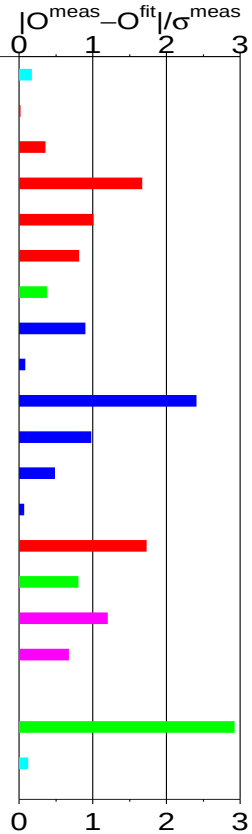
Status of the Standard Model

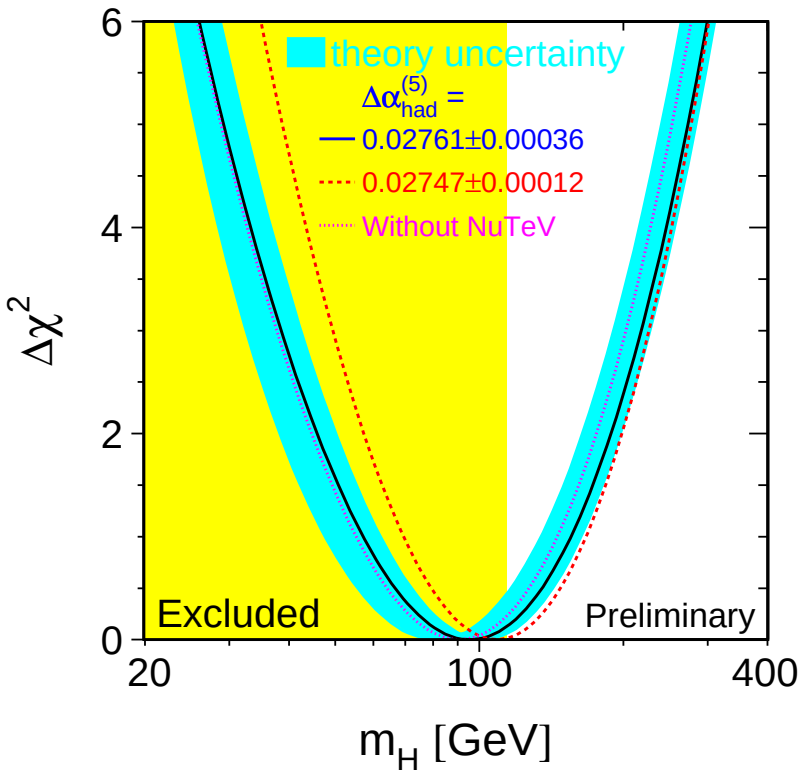
- **Precision data fit SM well and favor a Light Higgs:**

114.4 GeV (direct) $< M_H <$ 219 GeV (precision) **(95% C.L.)**

Summer 2003

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1036
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
$Q_W(Cs)$	-72.84 ± 0.46	-72.90





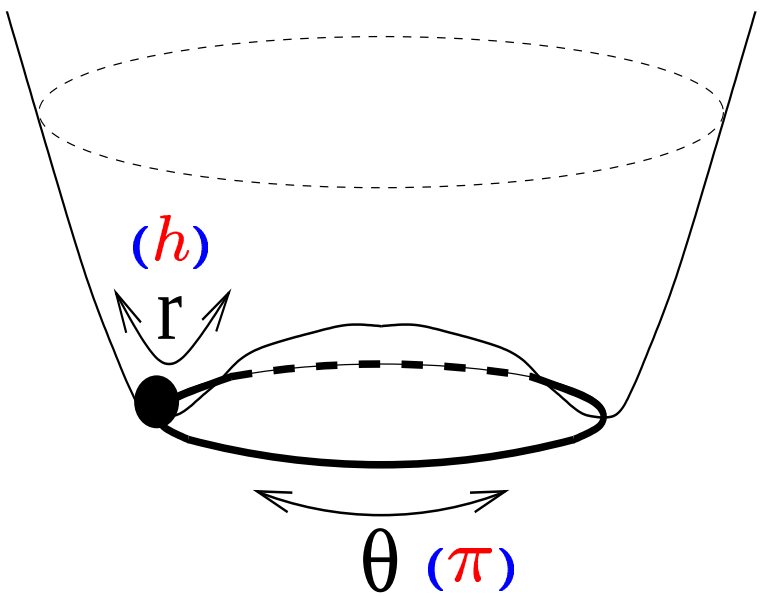
Higgs Mechanism & Higgs Mass in SM

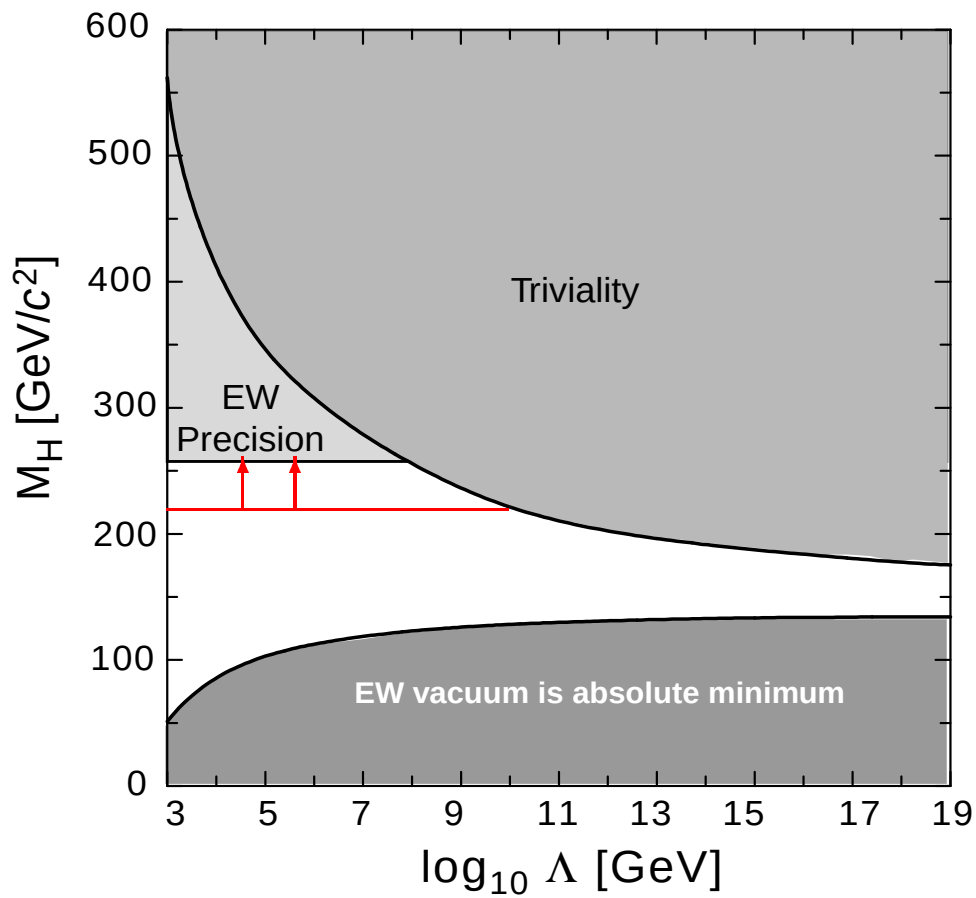
★ **SM Higgs Potential & Theory/Exp Constraints:**

$$V(H) = -\mu^2 H^2 + \lambda H^4, \qquad \Longrightarrow \qquad \langle H \rangle = \frac{v}{\sqrt{2}} = \sqrt{\mu^2/2\lambda}$$

$$M_H = \sqrt{2\lambda}\, v$$

$$v \simeq 250 \text{ GeV}$$

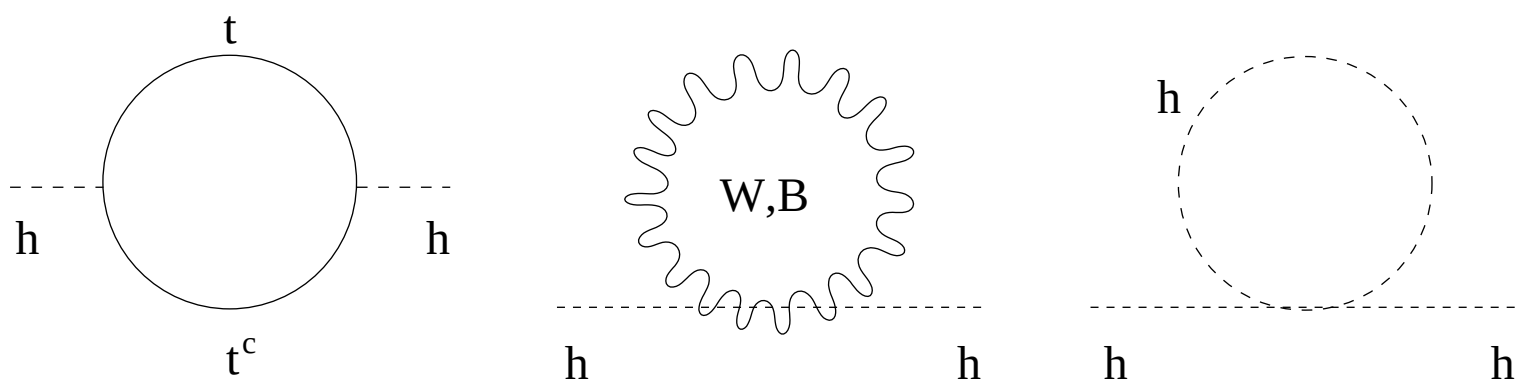




★ **SM with a light Higgs ($M_H \sim 160$ GeV) could be an Effective Theory valid up to Ultraviolet (UV) Cutoff $\Lambda \sim M_{\text{Planck}}$!! $\Rightarrow$ What's WRONG ??!**

Higgs Mass & Big Hierarchy Problem

▶ **BUT, SM Quantum Corrections are quadratically sensitive to Λ^2:**



(a) (b) (c)

$$M_H^2 = M_{H0}^2 - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

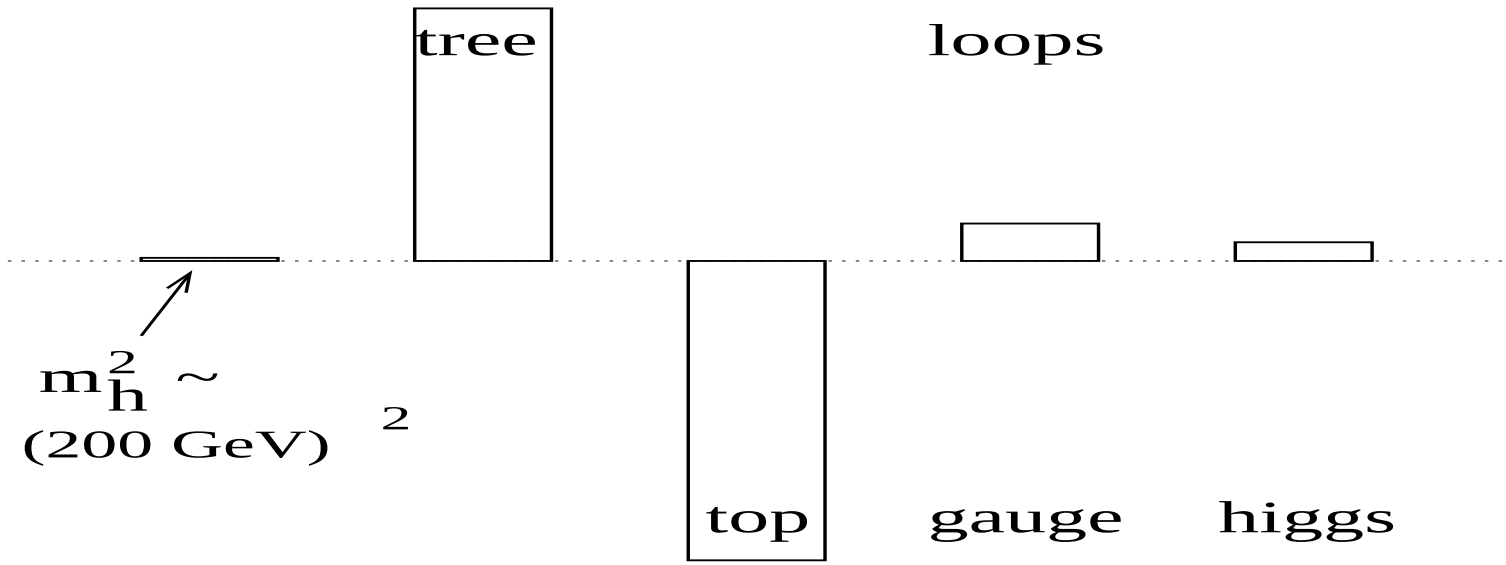
$$(200\,\text{GeV})^2 = M_{H0}^2 + \left[-(2\,\text{TeV})^2 + (700\,\text{GeV})^2 + (500\,\text{GeV})^2\right] \left(\frac{\Lambda}{10\,\text{TeV}}\right)^2$$

▶ **Big Hierarchy Problem:**

$\Lambda \sim M_{\text{Planck}}$ **requires a fine-tuned cancellation down to** 10^{-30} **!!!**

Higgs Mass & Little Hierarchy Problem

$$(200\,\text{GeV})^2 = M_{H0}^2 + \left[-(2\,\text{TeV})^2 + (700\,\text{GeV})^2 + (500\,\text{GeV})^2\right]\left(\frac{\Lambda}{10\,\text{TeV}}\right)^2$$



▶ **Little Hierarchy Problem:**

Demanding the fine-tuning $\gtrsim 10\%$ **in** M_H^2 **gives**

$$\Lambda_t \lesssim 3\,\text{TeV}, \qquad \Lambda_W \lesssim 9\,\text{TeV}, \qquad \Lambda_H \lesssim 12\,\text{TeV}$$

$\Rightarrow$ **New Phys below** $\sim$**3TeV for SM holding up to** $\sim$**10TeV !!!**

"Little Higgs" solves "Little Hierarchy"

- "Little Higgs" (LH) opens up a New Avenue for

Natural Electroweak Symmetry Breaking!

Arkani-Hamed, Cohen, Georgi, hep-ph/0105239

Arkani-Hamed, et al, hep-ph/0206020

Arkani-Hamed, et al, hep-ph/0206021

followed up by $O(100)$ papers since 2002-2003..... $(cf.\ Spires)$

- LH is an Effective Theory:

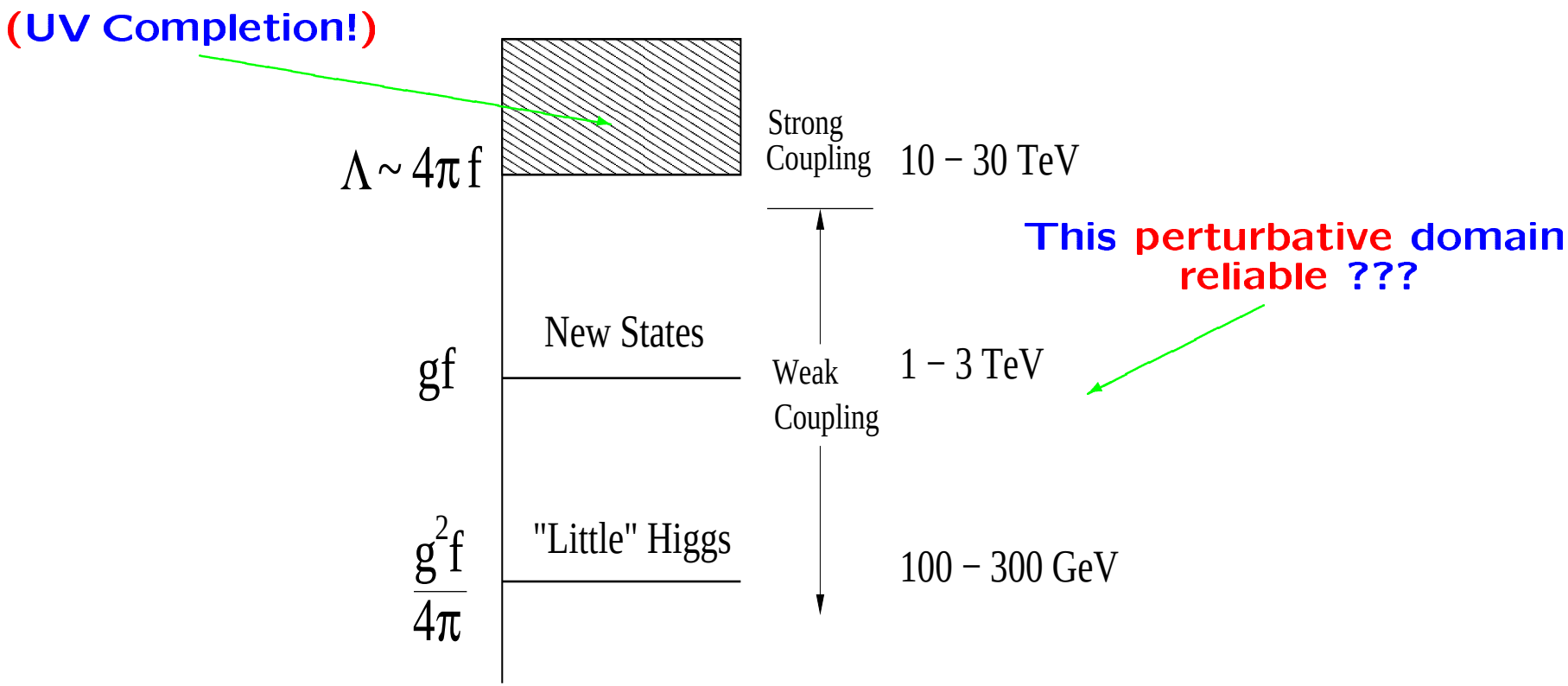
Weakly Coupled up to $\sim$10TeV

- Today I'll discuss something new about this direction ...

"Little Higgs" solves "Little Hierarchy"

- Higgs is a pseudo-Goldstone boson due to Global Symmetry Breaking, $\mathcal{G} \to \mathcal{H}$, at Ultraviolet (UV) Scale $\Lambda \sim 4\pi f$. *(Georgi & Pais, 1974)*
- Higgs is naturally Light as protected by Goldstone theorem; Higgs acquires small mass radiatively. Quadradic Divergence cancelled at 1-Loop by New States with Same Spin: *(Arkani-Hamed, et al, 2001,2002)*

$$W, Z, B \leftrightarrow W', Z', B', \qquad t \leftrightarrow t', \qquad H \leftrightarrow \chi, \Phi.$$



Little Higgs Models vs Unitarity

Chang & He, hep-ph/0311177

- **Our Observation** is that Global Symmetry Breaking of LH Model results in a **Large Multiplet of Many Goldstones** (including SM Higgs). The **collective effect** of Goldstone Scatterings will much enhance the S-matrix via Coupled channels, and thus strengthen Unitarity Limit $\Lambda_{\rm U}$ to be significantly below naive cutoff scale $\Lambda \sim 4\pi f$, ie, $\Lambda_{\rm U} \ll \Lambda$.

- **Global Symmetry Breaking Pattern:**
  $\mathcal{G} \to \mathcal{H}$ resulting in $\mathcal{N}$ Goldstones, π^a $(a = 1, 2, \cdots, \mathcal{N})$, which have the leading Lagrangian $(\Sigma \equiv \exp\left[i2\pi^a T^a/f\right])$,

$$\mathcal{L}_{\rm KE} = \frac{f^2}{8} {\rm Tr}\left[\partial^\mu \Sigma\, \partial_\mu \Sigma^\dagger\right]$$

- **Coupled Channel Analysis:**

  For the normalized $\mathcal{H}$ singlet state, $|S\rangle = \sum_{a=1}^{\mathcal{N}} \frac{1}{\sqrt{2\mathcal{N}}} |\pi^a \pi^a\rangle$, we derive the amplitude

$$T\left[S \to S\right] = \frac{\mathcal{C}}{\mathcal{N} f^2}\, s, \qquad \mathcal{C} = \sum_{a,c=1}^{\mathcal{N}} {\rm Tr}\left[T^a T^a T^c T^c - T^a T^c T^a T^c\right].$$

Unitarity of Little Higgs Models

Chang & He, hep-ph/0311177

- **Unitarity of S-matrix:** $S^\dagger S = 1$, $(S = 1 + iT)$ $\Rightarrow$ $T^\dagger T = 2\,\mathrm{Im}T$

- **Partial Wave Expansion:** $T = 16\pi \sum (2j+1) P_j(\cos\theta)\, a_j$

- **Unitarity Condition on Partial Wave:** $|\mathrm{Re}\, a_j| \;<\; 1/2$

- **Unitarity Limit of LH models:**

$$\sqrt{s} \;<\; \Lambda_{\mathrm{U}} = \sqrt{\frac{8\pi\mathcal{N}}{|\mathcal{C}|}}\, f$$

where $\mathcal{C} = \sum_{a,c=1}^{\mathcal{N}} \mathrm{Tr}\,[T^a T^a T^c T^c - T^a T^c T^a T^c]$.

- **Note $\Lambda_{\mathrm{U}} \ll 4\pi f$ for large $\mathcal{N}$ in LH models as Λ_{U} scales like $\Lambda_{\mathrm{U}} \propto 4\pi f\, O(1)/\mathcal{N}^{1/4}$.**

Unitarity of Littlest Higgs Model $SU(5)/SO(5)$

Chang & He, hep-ph/0311177

- **Littlest Higgs Model:** $\mathcal{N} = 14$ **Goldstones, under** $SU(2)_W \otimes U(1)_Y$**,**

$$\mathbf{1}_0\,(\chi_y^0) \;\oplus\; \mathbf{3}_0\,(\chi^{0,\pm}) \;\oplus\; \mathbf{2}_{\pm 1/2}\,(H) \;\oplus\; \mathbf{3}_{\pm 1}\,(\Phi)$$

- **Nonlinear field** Σ **is given by** $\Sigma = \exp\left[i2\Pi/f\right]\Sigma_0$ **with**

$$\Pi = \begin{pmatrix} \frac{1}{2}X & \frac{1}{\sqrt{2}}H^\dagger & \Phi^\dagger \\ \frac{1}{\sqrt{2}}H & \frac{2}{\sqrt{5}}\chi_y^0 & \frac{1}{\sqrt{2}}H^* \\ \Phi & \frac{1}{\sqrt{2}}H^T & \frac{1}{2}X^* \end{pmatrix}, \qquad X = \begin{pmatrix} \chi^0 - \frac{\chi_y^0}{\sqrt{5}} & \sqrt{2}\chi^+ \\ \sqrt{2}\chi^- & -\chi^0 - \frac{\chi_y^0}{\sqrt{5}} \end{pmatrix}, \quad \Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix}.$$

- **Summary of Unitarity Limits: with ALL 14 fields** $\Rightarrow$ $\boxed{\Lambda_{\rm U} = 3.17 f}$**, and**

Subsystem	$\Lambda_{\rm U}$	Subsystem	$\Lambda_{\rm U}$	Subsystem	$\Lambda_{\rm U}$	Subsystem	$\Lambda_{\rm U}$
$\{H\}$:	$5.79f$	$\{\chi^a\}$:	$5.01f$	$\{\Phi\}$:	$4.09f$		
$\{H, \chi^a\}$:	$4.35f$	$\{H, \Phi\}$:	$3.69f$	$\{\chi^a, \Phi\}$:	$3.45f$		
$\{H, \chi^a, \chi_y^0\}$:	$3.71f$	$\{H, \chi_y^0, \Phi\}$:	$3.45f$	$\{\chi^a, \chi_y^0, \Phi\}$:	$3.45f$	$\{H, \chi^a, \Phi\}$:	$3.27f$

Unitarity of Little Higgs vs UV Completion

Chang & He, hep-ph/0311177

- We found Unitarity Violation Scale $\Lambda_U \sim (3-4)f \ll \Lambda \sim 4\pi f \sim 10\,\text{TeV}$.

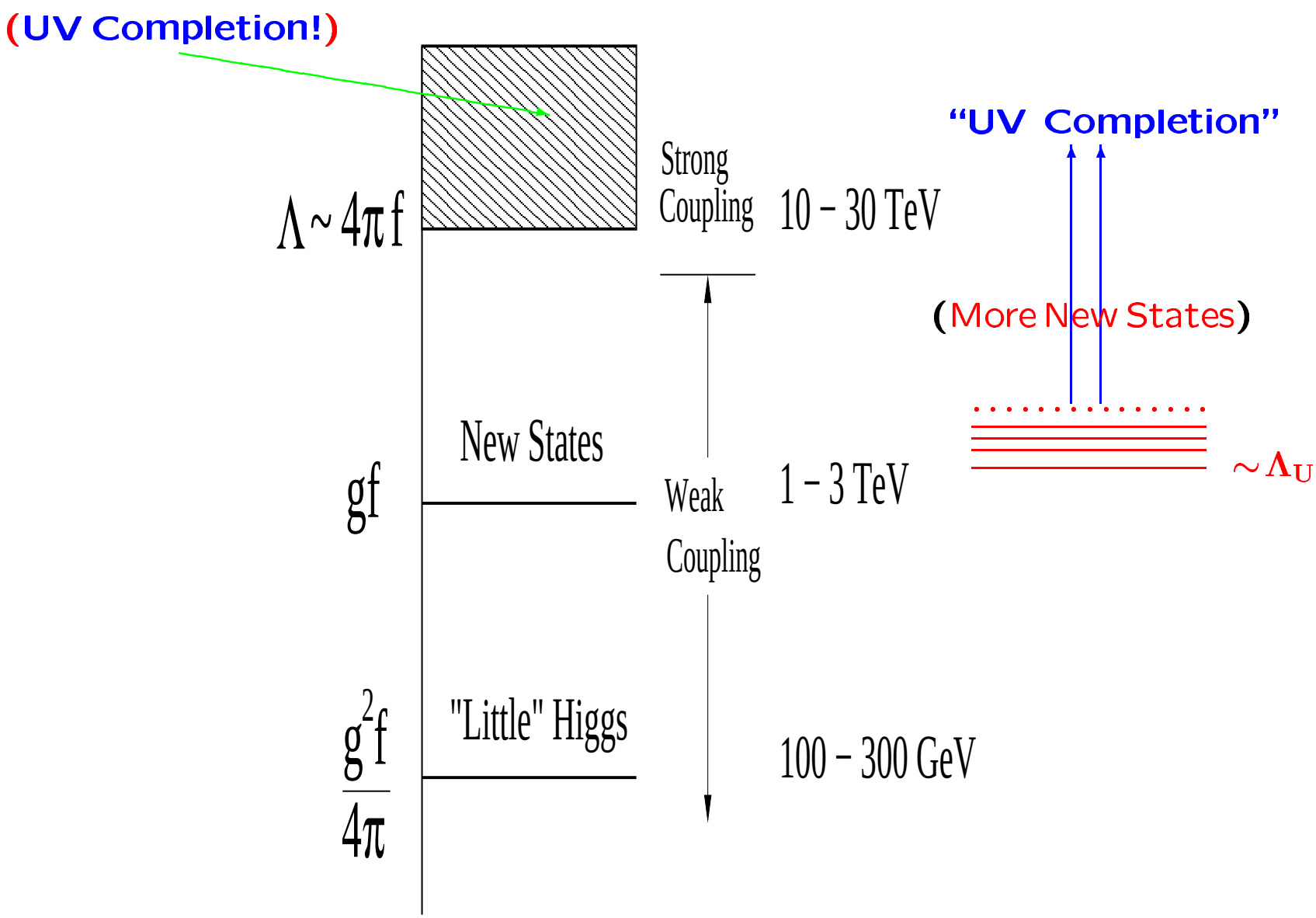
- Λ_U is comparable with W' mass $M_{W'}$ $\Rightarrow$ Crucial to do UV completion – additional New States must be included in the Multi-TeV range to restore Unitarity! $\Rightarrow$ More New Signals for LHC !

- New States reflect Little Higgs UV dynamics, and should be included in a way consistent with Λ^2-Cancellation in Higgs Mass at 1-Loop.

- Summary of Unitarity Bounds in various Models. (Note: $\mathcal{N} \gg 1$)

Little Higgs Model	G	H	$\mathcal{N}$	Λ_U/f	$M_{W'}/f$
Minimal Moose	$SU(3)^2$	$SU(3)$	8	2.89	2.29
Littlest Higgs	$SU(5)$	$SO(5)$	14	3.17	1.62
Antisymm Condensate	$SU(6)$	$Sp(6)$	14	3.68	1.62
$SO(5)$ Moose	$SO(5)^2$	$SO(5)$	10	4.09	3.24
$SO(9)$ Littlest Higgs	$SO(9)$	$SO(5) \otimes SO(4)$	20	3.79	2.29

Little Higgs: Role of UV Completion



Unitarity of Little Higgs: More Implications

Chang & He, hep-ph/0311177

- **Generalized diemsnional Analysis (GDA):** *(Chivukula et al, 92; Georgi, 93)*

$$\Lambda_{\rm GDA} \lesssim \min\left(\frac{a}{\sqrt{N_c}}, \frac{b}{\sqrt{N_f}}\right) 4\pi f < \Lambda_{\rm NDA} \simeq 4\pi f$$

  motivated by analysis of QCD-like TC dynamics with color N_c & flavor N_f. $\Rightarrow$ Note $\Lambda_{\rm GDA}$ and $\Lambda_{\rm U}$ have similar scalings, and are consistent.

- **$\Lambda_{\rm U}$ vs Precision Bounds:**

  Latest precision limits showed $f \gtrsim 0.5-1$TeV region is viable in most LH models *(cf. Chang, hep-ph/0306034, Peskin, et al, hep-ph/0310039.....)*

  (1) Cutoff of effective theory will be lowered to $\Lambda_{\rm U}$ (or $\Lambda_{\rm GDA}$);

  (2) More new states will contribute to precision observables; reanalysis with UV-completion is important.

  $\Rightarrow$ More detailed works are encouraged.....

- **$\Lambda_{\rm U}$ vs LHC & LC Signals:**

  - **Good chance for LHC to probe aspects of UV-completion, especially when $f \lesssim 1$TeV or some new states X are relatively light, $M_X \lesssim 1-3$ TeV.**

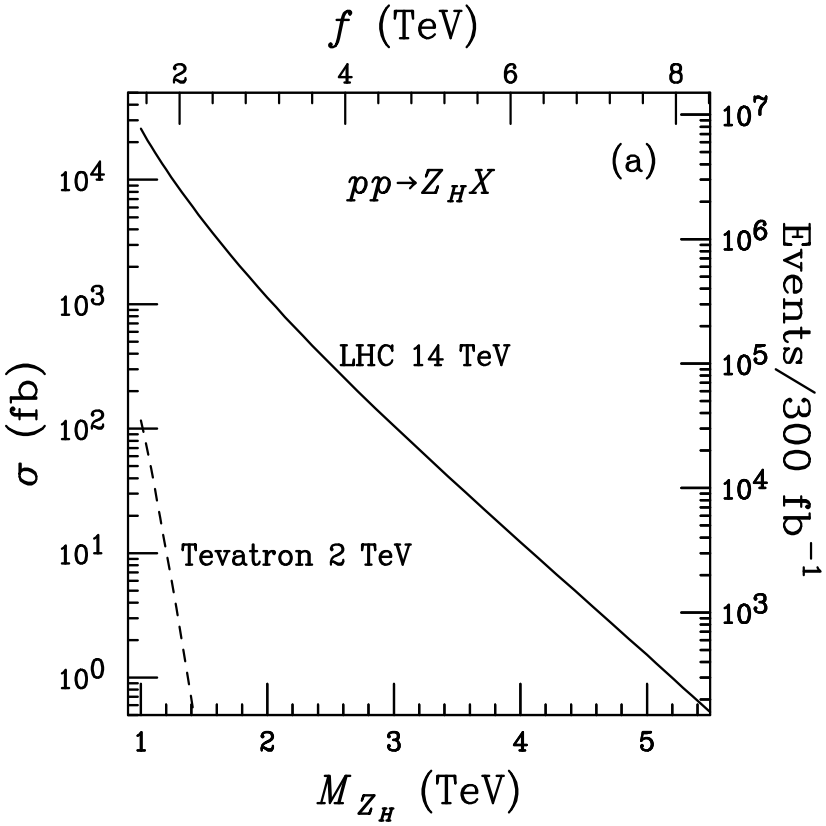
  - **New states X may mix with W', Z', χ, Φ etc, so that their effects can be carefully tested at LCs via measuring masses and couplings of lighter h, W', Z', etc.**

Implications for Collider Signals

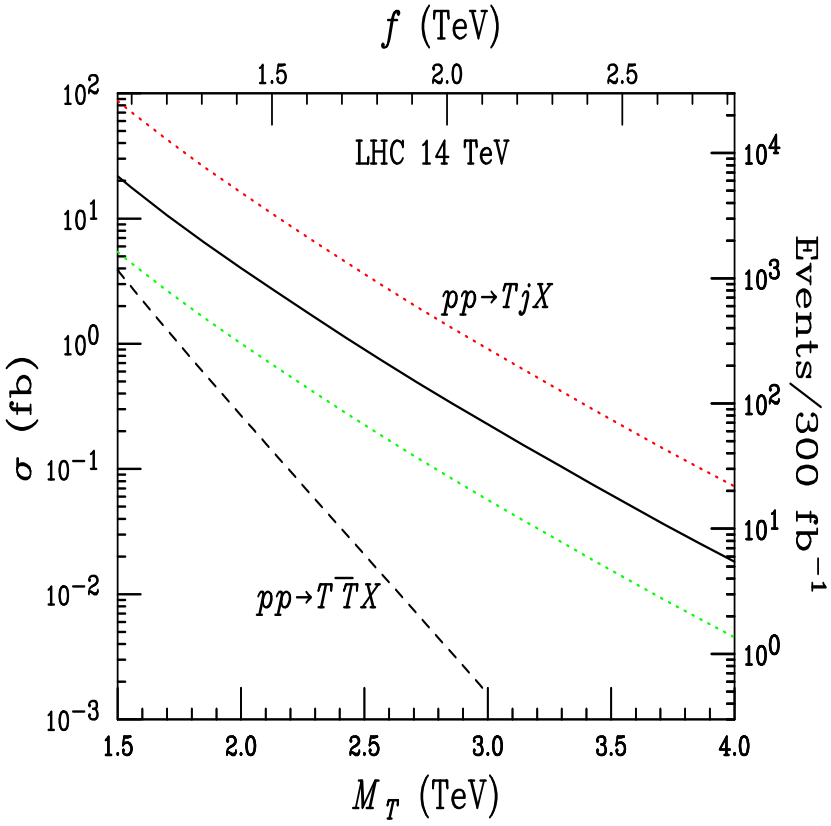
- **Typical Hadron Collider Signals of New States in Littlest Higgs Model.**

  (see: Han, et al, hep-ph/0301040; Peskin, et al, hep-ph/0310039; .....)

Z' Production



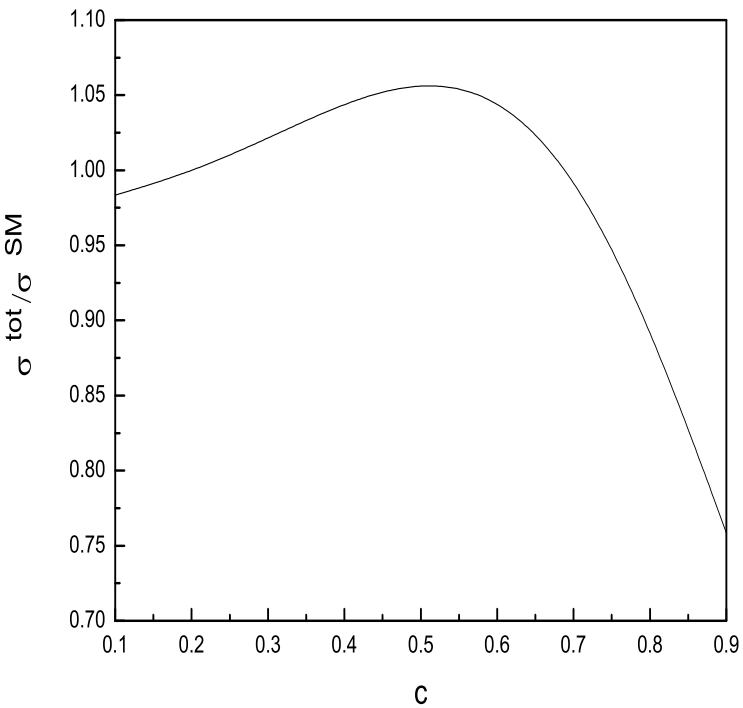
t' Production

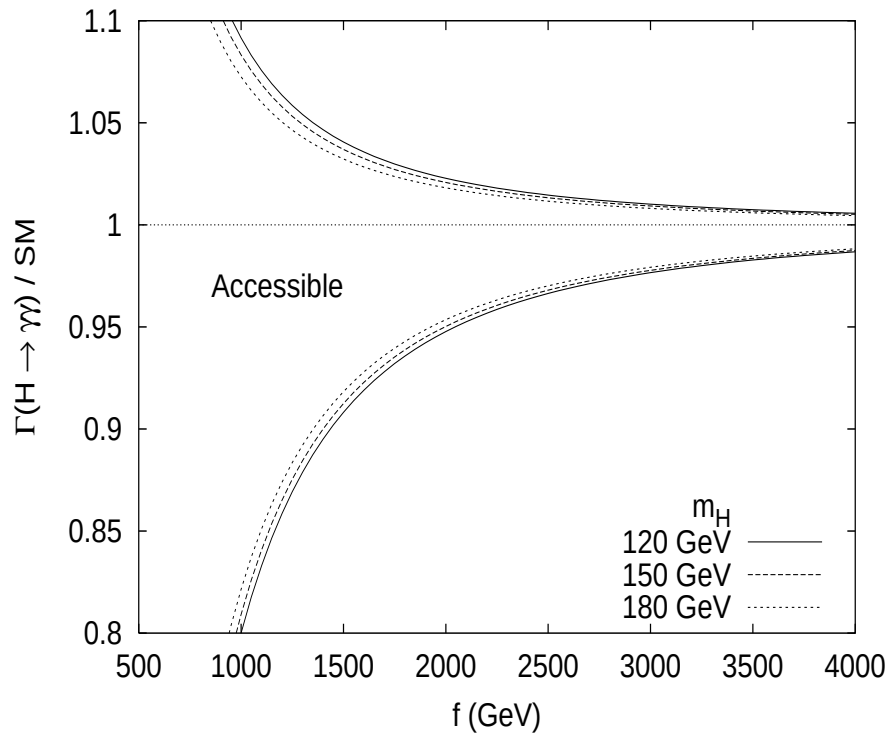


Implications for Collider Signals

- **Linear Collider: testing Littlest Higgs couplings to ZZ and to WW & $t\bar{t}$ at LCs.** *(e.g., Yue et al, hep-ph/0309113, Han et al, hep-ph/0302188, ...)*

$e^-e^+ \to Zh^0$ **($f = 2$TeV, $c \equiv g_1/\sqrt{g_1^2 + g_2^2}$).** $H \to \gamma\gamma$





- **For $f = 1$TeV, $\Gamma[H \to \gamma\gamma]$ varies within $+10\%$ & -20% relative to SM.**

- **$\gamma\gamma$ Collider: s-channel production cross section of $\gamma\gamma \to H$ $(\to b\bar{b})$ is proportional to $\Gamma[H \to \gamma\gamma]$. Deviation in $\Gamma[H \to \gamma\gamma]$ can be tested up to $f \lesssim 2.7\,(1.8)$ TeV at $1\,(2)\,\sigma$ level.**

Summary & Implications

▶ "Little Higgs" is an elegant idea, realizing Higgs as a Pseudo-Goldstone boson (–naturally light). It solves a Little Hierarchy Problem by ensuring the Λ^2-Cancellation among same spin fields at 1-Loop.

★ We observed that Global Symmetry Breaking of LH Model results in a Large Multiplet of Many Goldstones (including SM Higgs). The collective effect of Goldstone Scatterings much enhance the S-matrix via Coupled channels, and thus strengthen Unitarity Limit $\Lambda_{\rm U}$ to be significantly below naive cutoff scale $\Lambda \sim 4\pi f$, ie, $\Lambda_{\rm U} \ll \Lambda$.

★ Point to New Direction for Little Higgs Theory+Phenomenology studies:

▶ We showed the UV Completion is crucial and additional New States in Multi-TeV Range are forced by Unitarity.

⇒ New Signals for LHC & LC (& possible VLHC) !

⇒ Also, modification of existing precision bounds...

★ Implications for Colliders Signatures:

(1). Direct production & decays of New States t', W', Z', χ^a, Φ^a etc, e.g., $pp \to t'jX$, $q\bar{q}/e^-e^+ - (Z') \to WW$, ...

(2). Precision tests of h^0 couplings to (W, Z, t) & (W', Z', t') at LCs: $e^-e^+ \to Zh^0$, $H\nu\bar{\nu}$, $\gamma\gamma \to h^0 \to b\bar{b}$, etc.

⇒ New Signal Analyses are encouraged/upcoming .....